

2003 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics

General Instructions

- Reading time 5 minutes
- Working time 3 hours
- Write using black or blue pen
- Board-approved calculators and templates may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks - 120

- Attempt Questions 1-10
- All questions are of equal value
- Start each question in a new writing booklet

Question 1 (12 marks) Use a separate writing booklet.

Marks

(a) Evaluate
$$2\pi\sqrt{\frac{T}{f}}$$
 where $T=0.02$ and $f=150.8$, correct to two significant figures.

(b) Find
$$\int (x+1) dx$$

2

(c) If
$$8^x = 8000$$
, find x correct to two decimal places.

2

(d) Differentiate
$$(3x^2+2)^3$$
, with respect to x.

2

(e) Solve
$$x(8-x) = 3x - 24$$

2

(f) Solve the pair of simultaneous equations:
$$2x - y - 7 = 0$$

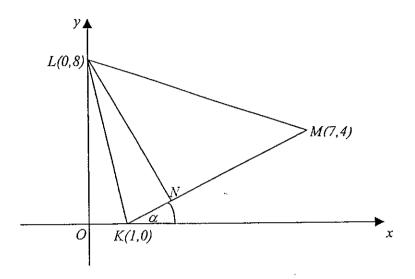
 $x + y + 1 = 0$

2

Question 2 (12 marks) Use a separate writing booklet.

Marks

In the diagram below, the points K, L, M have coordinates (1,0), (0,8) and (7,4) respectively and the angle between KM and the x-axis is α . The equation of the line LN is 3x + 2y = 3.



Copy this diagram into your writing booklet.

- (a) Find the gradient of KM and hence determine the size of the angle α , correct to the nearest degree.
- (b) Show that the equation of the line KM is 2x-3y-2=0.
- (c) Find the gradient of LN and hence show that LN is perpendicular to KM. 2
- (d) Show that the length of KM is $2\sqrt{13}$ units.
- (e) Find the exact perpendicular distance from L to KM.
- (f) Find the coordinates of P such that KLMP is a parallelogram.
- (g) Determine the exact area of *KLMP*.

Question 3 (12 marks) Use a separate writing booklet.

Marks

(a)
$$\frac{1}{5 - \sqrt{11}} = a + b\sqrt{11}.$$
 Find the value of a and b.

(b) Solve for real values of x

3

$$9^x - 12(3^x) + 27 = 0$$

(c) If $x = \log_e 2$ and $y = \log_e 3$, find $\log_e 6e$ in terms of x and y.

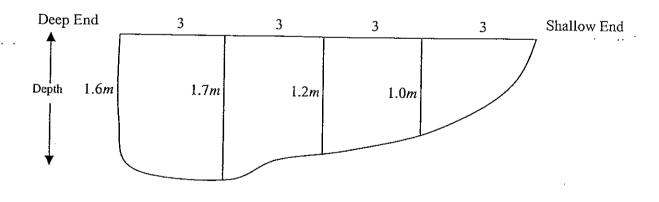
2

- (d) The diagram below shows the cross-section of a swimming pool from deep end to shallow end. The pool is 12 metres long and depths of the swimming pool are given at 3 metre intervals.
 - (i) Use Simpson's rule and 5 values to find an approximate value for the area of the cross-section.

3

(ii) The volume of water in the swimming pool is $80m^3$, find the width of the pool in metres, correct to 1 decimal place.

1



NOT TO SCALE

Question 4 (12 marks) Use a separate writing booklet.

Marks

(a) In a right triangle, find α , if $\sec \alpha = \csc 2\alpha$. (α and 2α are acute) 1

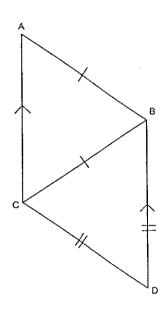
(b) (i) Show that $\sin^2 \theta - 3\cos^2 \theta = 4\sin^2 \theta - 3$

2

(ii) Hence, or otherwise, solve the equation, $\sin^2 \theta = 3\cos^2 \theta$ for $0^{\circ} \le \theta \le 360^{\circ}$.

3

(c)



In the diagram above, AB = BC, BD = CD and $AC \parallel BD$.

Copy the diagram into your writing booklet.

(i) Show that $\angle DBC = \angle BAC$ (giving reasons).

2

(ii) Prove that $\triangle ABC \parallel \triangle BDC$.

2

(iii) If AB = 7cm and BD = 4cm, find AC.

2

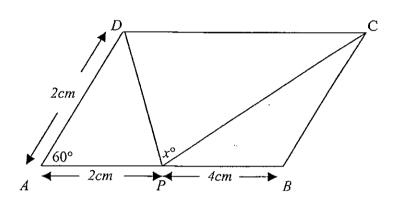
Question 5 (12 marks) Use a separate writing booklet.

Marks

(a) If
$$a(x-1)^2 + b(x-1) + c \equiv x^2$$
, find the values of a, b and c.

(b) Determine the equation of the normal to the curve $y = e^x + 2$ at x = 0.

(c)



NOT TO SCALE

In the figure, ABCD is a parallelogram in which AB = 6cm, AD = 2cm and $\angle DAB = 60^{\circ}$. The point P on AB is such that AP = 2cm and $\angle DPC = x^{\circ}$.

(i) Explain why the length of DP is 2 cm.

- 1
- (ii) Use the cosine rule in triangle PBC, to show that the length of PC is $2\sqrt{7}$. 2
- (iii) Hence, find the exact value for $\cos x^{\circ}$.

3

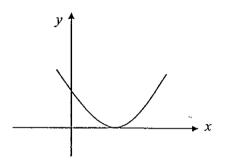
Question 6 (12 marks) Use a separate writing booklet.

Marks

3

(a) (i) For what values of k does the quadratic equation $(1+k)x^2 - 2kx - (3-2k) = 0$ 2 have reciprocal roots.

(ii) For any quadratic function the discriminant is either $\Delta = 0, \Delta < 0 \text{ or } \Delta > 0$. 2 State, which is the case for the function below. Hence describe the roots of the function.



- (b) Find $\int 2e^{\frac{x}{2}} dx$
- (c) The line y = x cuts the y-axis at A and the curve $y = x^3$ at B and C.
 - (i) On the same set of axes, sketch the two functions, and determine the co-ordinates of A, B and C.

(ii) Find the area bounded by the line and the curve.

Question 7 (12 marks) Use a separate writing booklet.

(a) (i) Find $\int \frac{1}{(3x-2)^3} dx$ 2
(ii) Give the exact value of $\int_1^3 \frac{dx}{3x-2}$ in simplest form.

2
(b) Find the equation of the curve which passes through the point (1, 2) and whose gradient at any point (x, y) is given by y' = 3x - 2.

(c) For the curve $y = x^3 - 3x + 5$:

- (i) Find the stationary points and determine their nature. 3
- (ii) Find the co-ordinates of the inflexion point.
- (iii) Is this function increasing or decreasing at x = 2?
- (iv) Sketch the curve.

Question 8 (12 marks) Use a separate writing booklet.

Marks

(a) The weekly wages of 100 employees in Virgin Red are listed in increasing order from cleaner to Managing Director, forming the sequence:

\$280, \$335, \$390,

(i) What is the highest pay per week?

2

(ii) Calculate the total payroll for one week.

1

- (b) A = (1, 1), B = (4, 7) and P = (x, y).
 - (i) Write expressions for the lengths of PA and PB.

1

2

- (ii) P moves so that $PA = 2 \times PB$. Show that the locus of P is the circle: $x^2 - 10x + y^2 - 18y + 86 = 0$.
- (iii) Find the centre and radius of this circle.

2

Question 8 (continued)

Marks

4

(c) Below are the graphs of y = f'(x) and y = f''(x) for the function y = f(x). Describe what is happening on the original curve y = f(x) at A and B. You must justify your answers.

y = f'(x) y = f''(x)

Question 9 (12 marks) Use a separate writing booklet.

Marks

- (a) (i) The fourth term of a geometric progression is 6 and the ninth term is $\frac{64}{81}$.

 By solving simultaneous equations show that the first term is $\frac{81}{4}$ and that the common ratio is $\frac{2}{3}$.
 - (ii) How many terms of this sequence must be added so that the sum exceeds 60?

- (b) (i) Differentiate $\log_e(\log_e x)$, with respect to x.
 - (ii) Hence, find the exact value of $\int_{e}^{e^{2}} \frac{dx}{x \log_{e} x}.$ 3

(c)

Question 10 (12 marks) Use a separate writing booklet.

Marks

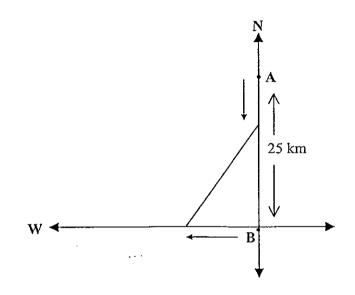
(a) Find the value of
$$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$$

(b)
$$y = \frac{1}{\sqrt{x}}$$
 is rotated about the x-axis from $x = 3$ to $x = a$.

The volume of the resulting solid is 2π cubic units. What is the exact value of a?

On the compass diagram below, Mary is at A, 25 km due north of position B.

John is at B. Mary walks towards B at 4 km/h. John moves due west at 6 km/h.



(i) Show that the distance between Mary and John after t hours is given by: 2

$$d^2 = 52t^2 - 200t + 625$$

(ii) Letting $L = d^2$, find the time when L is minimum.

(iii) Hence find the minimum distance between John and Mary, correct to the nearest kilometre.

End of Examination.



Mathematics Trial 2003 Marking Criteria

Question 1 (a)

Criteria	Marks
• Gives correct answer, 0.72	2
Gives correct answer but rounds incorrectly, OR	1
Correctly rounds incorrect simplification	•

Question 1 (b)

Criteria	Marks
• Gives correct answer, $\frac{x^2}{2} + x + c$	2
• correctly integrates either x or 1 (ignore constant of integration)	1

Question 1 (c)

Criteria	Marks
Gives correct answer, 4.32	2
Attempts to solve using logarithms but makes subsequ OR	ent error 1
Gives correct answer but fails to round correctly	

Question 1 (d)

. Criteria	Marks
• Gives correct answer, $18x(3x^2+2)^2$ or $18x(9x^4+12x^2+4)$	2
• Fails to find derivative of $(3x^2 + 2)$	1
Fails to use appropriate notation	

Question 1 (e)

Criteria	Marks
• Correctly finds both solutions for $x = 8, -3$	2
Sets up correct quadratics equation OR	. 1
 Finds only one solution 	

Question 1 (f)

Criteria	Marks
• Correctly finds that $x=2$ and $y=-I$	2
• Attempts a method of solution to isolate x or y OR	1
• Correctly solves for only one of x or y	

Question 2 (a)

Criteria	Marks
• Gives correct answer, $m = \frac{2}{3}$ $\theta = 34^{\circ}$	2
Finds correct gradient OR	1
• Finds correct θ , from incorrect gradient	

Question 2 (b)

	Criteria	Marks
Gives correct answer,	2x-3y-2=0	2
Correct substitution but mal	ces subsequent error	1

Question 2 (c)

Criteria	Marks
• Gives correct answer, $m = -\frac{3}{2}$ and shows gradients are negative reciprocals	2
• Finds correct gradient but fails to show perpendicularity.	1

Question 2 (d)

Criteria	Marks
• Uses distance formula with correct substitution to get $KM = 2\sqrt{13}$ units	1

Question 2 (e)

	Criteria	Marks
 Gives correct answer, per (ignore rationalising) 	pendicular dis tan $ce = 2\sqrt{13}$ units	2
 Uses correct formula but make 	s gubsaguant arra-	

Question 2 (f)

Criteria	Marks
• Gives correct answer, P(8, -4)	1

Question 2 (g)

<u>Criteria</u>	Marks
Gives correct answer, Area = 52 sq. units	2
 Recognises to use perpendicular height and distance KM, but makes subsequent error. OR Finds the area of only one triangle. Ignore units. 	1

Question 3 (a)

<u>Criteria</u>	Marks
• Gives correct answer, $a = \frac{5}{14}$, $b = \frac{1}{14}$	3
 Multiplies by correct conjugate and simplifies but fails to give a or b. OR	2 ·
 Obtains correct answer for either a or b. 	ľ

Question 3 (b)

Criteria	Marks
• Gives correct answer, $x = 2$, $x = 1$	3
 Obtains correct quadratic equation, but fails to find both values of x. OR Obtains correct x values with no working shown. 	2
 Obtains one correct value for x with no working shown. OR Obtains correct quadratic equation only. OR 	 I
• Finds values for "u" only.	

Question 3 (c)

Criteria	Marks
• Gives correct answer, $x+y+1$	2
 Rewrites logarithm as log_e 2 + log_e 3 + log_e e, but makes subsequent error. OR Obtains part of correct answer from correct expression. 	1

Question 3 (d)(i)

Criteria Criteria	Marks
• Gives correct answer, $Area = 14.8m^2$	3
• Finds correct "h" and uses correct substitutions but makes subsequent error. OR	2
• Finds correct "h" but makes one error in substitutions.	
Finds correct "h", uses incorrect formula.	1

Question 3 (d)(ii)

Criteria	Marks
 Gives correct answer, width = 5.4m. Ignore rounding. OR 	1
Finds correct width using incorrect answer from (i)	

Question 4 (a)

Criteria	Marks
• Gives correct answer, $\alpha = 30$.	1

Question 4 (b)(i)

Criteria	Marks
• Correctly substitutes $\cos^2 \theta = 1 - \sin^2 \theta$ and simplifies to find RHS.	. 2
Uses correct substitution but makes subsequent error OR	1
Uses poor setting out.	

Question 4 (b)(ii)

Criteria	Marks
• Gives all four correct answers 60°,120°,240°,300°.	3
• Finds two of the four answers by not finding $\sin \theta = \pm \frac{\sqrt{3}}{2}$	2
• Correctly rewrites equation as $4\sin^2\theta - 3 = 0$ but cannot go further.	1

Question 4 (c)(i)

Criteria	Marks
• States that $ \angle DBC = \angle BCA (alternate angles AC \parallel BD) $ $ \angle BAC = \angle BCA (base angles isosceles \Delta) $	2
No reasons given	1

Question 4 (c)(ii)

Criteria Criteria	Marks
Shows two pairs of corresponding angles equal.	2
Shows a pair of corresponding angles equal.	1

Question 4 (c)(iii)

Criteria	Marks
• Gives correct answer, $AC = 12.25cm$.	2
Uses correct corresponding sides but incorrect substitution OR	1
Uses correct corresponding sides but makes subsequent error	

Question 5 (a)

Criteria	Marks
• Gives correct answer, $a = 1, b = 2, c = 1$.	3
Gives two correct answers.	2
Gives one correct answer.	1

Question 5 (b)

Criteria	Marks
• Gives correct answer, $x + y - 3 = 0$.	3
 Finds gradient of normal but makes subsequent error in equation. OR Finds correct gradient of normal only. 	2
OR • Finds gradient of tangent and uses this to find equation.	
Finds gradient of tangent only.	1

Question 5 (c)(i)

Criteria	Marks
• Shows clearly why DP is 2 cm.	1

Question 5 (c)(ii)

Criteria	Marks
Gives correct substitution into correct cosine rule and shows working to obtain answer.	2
Uses incorrect substitution into correct cosine rule.	1

Question 5 (c)(iii)

3
2
1

Question 6 (a)(i)

Criteria	Marks
• Gives correct answer, $k = \frac{2}{3}$.	2
• Gives correct criterion, i.e. $\alpha\beta = 1$. and subsequent error.	1

Question 6 (a)(ii)

Criteria	Marks
• Gives correct answer, $\Delta = 0$ (1 mark), two real equal roots (1 mark).	2
Gives one or other of above.	1

Question 6 (b)

Criteria	Marks
• Gives correct answer, $4e^{\frac{x}{2}} + c$.	2
• Gives answer as $ke^{\frac{x}{2}} + c$, where $k \neq 4$ OR	
• Gives answer as $4e^{\frac{x}{2}}$, without "c".	

Question 6 (c)(i)

. Criteria	Marks
• Gives correct graph (1 mark) and values for $x = 0, -1, 1$ (1 mark) and $y = 0, -1, 1$ (1 mark) respectively.	3
Correct graph, incorrect with one point OR	2
• Incorrect graph, correct x and y values from graph	
 Correct graph, no x and y values given 	1
Incorrect graph for 1 function, some correct x and y values	

Question 6 (c)(ii)

Marks
3
2
1

Question 7 (a)(i)

Criteria	Marks
• Gives correct answer, $\frac{-1}{6(3x-2)^2} + c$ (ignore "c")	2
• Writes integral as $(3x-2)^{-3}$ OR	1
• Integrates $(3x-2)^{-3}$ but fails to divide by correct constant	

Question 7 (a)(ii)

Criteria	Marks
• Gives correct answer, $\frac{\ln 7}{3}$	2
Correct integral only	1
OR Correct decimal answer, 0.649	

Question 7 (b)

Criteria	Marks
• Gives correct answer, $y = \frac{3}{2}x^2 - 2x + \frac{5}{2}$	2
• Correct integral only, fails to find c	1

Question 7 (c)(i)

Criteria	Marks
• Gives correct answer, (1, 3) is a minimum turning point and (-1, 7) is a maximum turning point. With working to support answer. (ignore y values)	
Finds correct derivative, equates with zero, finds points without nature OR	2
Correctly finds only one point and its nature	
Finds correct derivative	1

Question 7 (c)(ii)

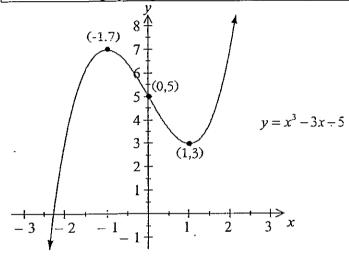
Criteria	Marks
• Finds correct inflexion point, (0,5)	1

Question 7 (c)(iii)

Criteria	Marks
Gives correct condition, function is increasing	1

Question 7 (c)(iv)

Criteria	Marks
Correct graph to go with answers to previous parts	1



Question 8 (a)(i)

Criteria	Marks
• Correct answer, \$5 725	2
• Correct value for $d = 55	1

Question 8 (a)(ii)

Criteria	Marks
Correct answer, \$300 250	L

Question 8 (b)(i)

	Criteria		Marks
Correct expressions,	$PA = \sqrt{(x-1)^2 + (y-1)^2},$	$PB = \sqrt{(x-4)^2 + (y-7)^2}$	1

Question 8 (b)(ii)

Criteria	Marks
• Correctly equates $PA = 2 \times PB$, correct expansion and simplification	2
• Correctly equating $PA^2 = 4 \times PB$, making subsequent error	1

Question 8 (b)(iii)

Criteria	Marks
• Correct answer, centre $(5,9)$ and radius = $2\sqrt{5}$ units	2
 Correctly finding centre or radius OR 	1
Correctly completing the square on both variables	<u> </u>

Question 8 (c)

Criteria	Marks
• Correct answer: At A, $y' = 0$, and $y'' < 0$: max imum stationary point At B, $y' > 0$ and $y'' = 0$,: point of inf lexion and y'' changes sign	4
Gives 3 out of 4 criteria	3
 Gives minimum turning point at A and point of inflexion at B with no support OR 	2
• Gives At A, $y' = 0$, and $y'' < 0$ At B, $y' > 0$ and $y'' = 0$, with only one conclusion	
• Finds only A or B	1

Question 9 (a)(i)

Criteria	Marks
Correctly setting up simultaneous equations and solving showing all working	3

Correctly setting up simultaneous equations then making subsequent error	2
• Giving $T_4 = 6$ and $T_9 = \frac{64}{81}$ without further solution.	I

Question 9 (a)(ii)

Criteria	Marks
• Giving correct answer, $n = 11$, showing all necessary working	4
 Finding that n > 10.8, with correct working Finding that n < 10.8, with correct working and stating n = 10 	3
• Obtaining $\frac{1}{81} > (\frac{2}{3})^n$ then not using logarithms or trial and error to find n	2
• Correct substitution into correct formula, i.e. $S_n = \frac{a(1-r^n)}{1-r}$ $\frac{\frac{81}{4}(1-\frac{2}{3})}{1-\frac{2}{3}} > 60$	1
13	

Question 9 (b)(i)

Criteria	Marks
• Correct answer: $y' = \frac{1}{x \log_e x}$, with correct notation	2
S _E	1

• Giving answer as $y' = \frac{1}{\log_e x}$	
OR Se	
Giving correct answer without correct notation	

Question 9 (b)(i)

Criteria	Marks
• Correct answer: $\log_e 2$, with correct notation	3
• Giving correct answer that is not exact, 0.693 OR	2
• Giving correct integral and substitution, e.g. $\log_e 2 - \log_e 1$	
 Giving correct integral, log_e(log_e x) 	

Question 10 (a)

Criteria	Marks
• Correct answer: $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{2}$, with working	2
• Giving $a = \frac{1}{3}$ and $r = \frac{1}{3}$	I

Question 10 (b)

Criteria	Marks
• Correct answer: $a = 3e^2$, with all working	4
• Gives decimal answer: $a = 22.17$ OR • Equates $\pi \int_{3}^{a} \frac{1}{x} dx = 2\pi$ and integrates correctly but makes subsequent error	3
• Finds $y^2 = \frac{1}{x}$ and equates $\pi \int_{3}^{a} \frac{1}{x} dx = 2\pi$	2

Question 10 (c)(i)

Criteria	Marks
 Correctly uses Pythagoras' theorem with Mary's distance (25-4t) and John's distance (6t), to obtain answer 	2
Uses Pythagoras' theorem but uses incorrect distance for Mary or John OR	1
Uses correct distances but does not show enough working	

Question 10 (c)(ii)

Criteria	Marks
• Finds correct time, $\frac{25}{12}$ = 1.92 hours and includes check of minimum	3
• Finds $\frac{dL}{dt} = 104t - 200$, correctly finds t but does not check minimum	2
Finds incorrect value for t from correct derivative and checks minimum	
• Finds $\frac{dL}{dt} = 104t - 200$, does not go further	1

Question 10 (c)(iii)

Criteria	Marks
Finds correct distance, 21 kilometres (ignore rounding) OR	1
Finds correct distance from previous error	
,	